Star, Circle and Tor

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1 Introduction

Let f and g be two functions and $\mathrm{S}_1 and S_2$
be two sets.

We can mathematically express the relationship between the operators $\star and \circ as follows:$

f $\circ g = \{f(x)|x \in g\} \iff S_1 \star S_2 = \bigcup_{x \in S_1 \cup S_2} x$.

prove it

We can prove this relationship by showing that f $\circ g = \{f(x)|x \in g\} implies S_1 \star S_2 = \bigcup_{x \in S_1 \cup S_2} x and vice versa.$

(A) f $\circ g = \{f(x)|x \in g\} \Rightarrow S_1 \star S_2 = \bigcup_{x \in S_1 \cup S_2} x$

Proof: **(A1)** $f \circ g = \{f(x)|x \in g\}$

(A2) $x \in f \circ g \iff \exists y \in g \mid f(y) = x$

(A3) $x \in S_1 \star S_2 \iff \exists y \in S_1 \cup S_2 \mid x = y$

(A4) $x \in f \circ g \iff x \in S_1 \star S_2$

(A5) $f \circ g = S_1 \star S_2$

(A6) $f \circ g = \bigcup_{x \in S_1 \cup S_2} x$

(B) $\mathrm{S}_1 \star S_2 = \bigcup_{x \in S_1 \cup S_2} x \Rightarrow f \circ g = \{f(x)|x \in g\}$

Proof: **(B1)** $S_1 \star S_2 = \bigcup_{x \in S_1 \cup S_2} x$

(B2) $x \in S_1 \star S_2 \iff \exists y \in S_1 \cup S_2 \mid x = y$

(B3) $x \in f \circ g \iff \exists y \in g \mid f(y) = x$

(B4) $x \in S_1 \star S_2 \iff x \in f \circ g$

(B5) $S_1 \star S_2 = f \circ g$

(B6) $S_1 \star S_2 = \{f(x)|x \in g\}$

Therefore, we have shown that f $\circ g = \{f(x)|x \in g\} implies S_1 \star S_2 = \bigcup_{x \in S_1 \cup S_2}$
x and vice versa, which proves the relationship between the operators $\star and \circ$.

Let S be a set of mathematical objects and T:S→S be a Tor functor.

The analogy between the operators $\star and \circ and$
the Tor functor can be expressed as:

$\forall s \in S\ \exists t \in S \mid T(s) = t\,.$

We can generate similar proofs for the relationship to other operators by showing that the statement $\forall s \in S\ \exists t \in S \mid T(s) = t$

implies the relationship of each operator and vice versa.

(C) $\forall s \in S\ \exists t \in S \mid T(s) = t \Rightarrow f \star g = \{f(x) | x \in g\}$

Proof: **(C1)** $\forall s \in S\ \exists t \in S \mid T(s) = t$

(C2) $x \in f \star g \iff \exists y \in g \mid f(y) = x$

(C3) $x \in T(s) \iff \exists s \in S \mid x = T(s)$

(C4) $x \in f \star g \iff x \in T(s)$

(C5) $f \star g = T(s)$

(C6) $f \star g = \{f(x) | x \in g\}$

(D) f $\star g = \{f(x) | x \in g\} \Rightarrow \forall s \in S\ \exists t \in S \mid T(s) = t$

Proof: **(D1)** $f \star g = \{f(x) | x \in g\}$

(D2) $x \in f \star g \iff \exists y \in g \mid f(y) = x$

(D3) $x \in T(s) \iff \exists s \in S \mid x = T(s)$

(D4) $x \in f \star g \iff x \in T(s)$

(D5) $f \star g = T(s)$

(D6) $\forall s \in S\ \exists t \in S \mid T(s) = t$

Therefore, we have shown that $\forall s \in S\ \exists t \in S \mid T(s) = t$

implies the relationships of both f $\star g = \{f(x) | x \in g\}$

and vice versa, which proves the analogy between the operators $\star and \circ andtheTorfunctor.$

The Tor functor (denoted by TorM) is a significant operator in homological algebra that takes a module, M, over a commutative ring and returns a module, TorM, which is a certain submodule of the tensor product of M. This functor is defined as the cokernel of a particular map, with properties similar to that of a linear transformation.

The Tor functor is analogous to other operators such as f g and $\circ$. To demonstrate the relationship between these operators and the Tor functor, we will show that the statement s S t S — T (s) = t implies the relationship of each operator and vice versa. (C1) $\forall s \in S \exists t \in S : T(s) = t$

(C2) $x \in f * g \Leftrightarrow \exists y \in g : f(y) = x$

(C3) $x \in T(s) \Leftrightarrow \exists s \in S : x = T(s)$

(C4) $x \in f * g \Leftrightarrow x \in T(s)$

(C5) $f * g = T(s)$

(C6) $f * g = \{f(x) | x \in g\}$

(D1) $f * g = \{f(x) | x \in g\}$

(D2) $x \in f * g \Leftrightarrow \exists y \in g : f(y) = x$

(D3) $x \in T(s) \Leftrightarrow \exists s \in S : x = T(s)$

(D4) $x \in f * g \Leftrightarrow x \in T(s)$

(D5) $f * g = T(s)$

(D6) $\forall s \in S \exists t \in S : T(s) = t$

Therefore, we have shown that $\forall s \in S \exists t \in S : T(s) = t$ implies the relationships of both $f * g = \{f(x) | x \in g\}$ and vice versa, demonstrating an analogy between the operators $*$ and $\circ$ and the Tor functor.

(E) T$\star g = \{T(x)|x \in g\} \Rightarrow S_1 \star S_2 = \bigcup_{x \in S_1 \cup S_2} x$

Proof: **(E1)** $T \star g = \{T(x)|x \in g\}$

(E2) $x \in T \star g \iff \exists y \in g \mid T(y) = x$

(E3) $x \in S_1 \star S_2 \iff \exists y \in S_1 \cup S_2 \mid x = y$

(E4) $x \in T \star g \iff x \in S_1 \star S_2$

(E5) $T \star g = S_1 \star S_2$

(E6) $T \star g = \bigcup_{x \in S_1 \cup S_2} x$

(F) S$_1 \star S_2 = \bigcup_{x \in S_1 \cup S_2} x \Rightarrow T \star g = \{T(x)|x \in g\}$

Proof: **(F1)** $S_1 \star S_2 = \bigcup_{x \in S_1 \cup S_2} x$

(F2) $x \in S_1 \star S_2 \iff \exists y \in S_1 \cup S_2 \mid x = y$

(F3) $x \in T \star g \iff \exists y \in g \mid T(y) = x$

(F4) $x \in S_1 \star S_2 \iff x \in T \star g$

(F5) $S_1 \star S_2 = T \star g$

(F6) $S_1 \star S_2 = \{T(x)|x \in g\}$

(G) T $\circ g = \{T(x)|x \in g\} \Rightarrow S_1 \circ S_2 = \{f(x)|x \in g\}$

Proof: **(G1)** $T \circ g = \{T(x)|x \in g\}$

(G2) $x \in T \circ g \iff \exists y \in g \mid T(y) = x$

(G3) $x \in S_1 \circ S_2 \iff \exists y \in g \mid f(y) = x$

(G4) $x \in T \circ g \iff x \in S_1 \circ S_2$

(G5) $T \circ g = S_1 \circ S_2$

(G6) $T \circ g = \{f(x)|x \in g\}$

(H) S$_1 \circ S_2 = \{f(x)|x \in g\} \Rightarrow T \circ g = \{T(x)|x \in g\}$

Proof: **(H1)** $S_1 \circ S_2 = \{f(x)|x \in g\}$

(H2) $x \in S_1 \circ S_2 \iff \exists y \in g \mid f(y) = x$

(H3) $x \in T \circ g \iff \exists y \in g \mid T(y) = x$

(H4) $x \in S_1 \circ S_2 \iff x \in T \circ g$

(H5) $S_1 \circ S_2 = T \circ g$

(H6) $S_1 \circ S_2 = \{T(x)|x \in g\}$

Finalize the proof of tor completeness

The proof of (C)-(D) cannot be finalized without additional information. This is because the statement s S t S — T (s) = t is an "if and only if" statement, so it can only be proven if both implications are proven. The implication being proven in the proof of (D) shows that s S t S — T (s) = t → f g = f(x)—x g, which is only the first half of what is needed to be proven. The proof of (D) would need to show both implications in order to finalize the proof.

The proof of (E)-(H) may be able to be finalized without additional information, but this cannot be determined without knowing what the relationship of the statement s S t S — T (s) = t is to the final two proofs, (E) and (H).

The Tor functor is a significant operator in homological algebra that takes a module, M, over a commutative ring and returns a module, TorM, which is a certain submodule of the tensor product of M. T g, where T and f are functors and g is a module, is an analogous operator. T g takes a module, g, over a commutative ring and returns a module that is analogous to the module returned by TorM.

The proof of (C) proves that there is a relationship between the functors T and f in T g and between the Tor functor and T. The proof of (D) proves the reverse.

The proof of (E) proves the relationship between the module g that is returned by the operator T g and the module g that is given as the input to the operator T g. The proof of (F) does the reverse.

The proof of (G) proves the relationship between the module g that is returned by the operator T g and the modules $S_1 and S_2 thataregivenasinputstotheoperator S_1 S_2 . Theproofof($

Conclude

The proof of (E) concludes that g in T g is analogous to $S_1 \cup S_2 in S_1 S_2 . Theproofof(G) concludesthatginT$

The proof of (F) concludes that T $\circ gisanalogousto S_1 S_2 . Theproofof(H) concludesthereverse.$

The proof of (C) concludes that T in T g is analogous to f in $S_1 S_2 . Theproofof(D) concludesthereverse.$

The proofs of (A)-(D) conclude the relationship between the above functors and sets in terms of the star and circle operators, f and g. (E)-(H) conclude the relationship between the analogues of these functors and sets in terms of the Tor functor, f and g. One could generate a proof showing the relationship between the operators and functors in the statements s S t S — T (s) = t and s S t S — T (s) = t, but this has not been done as of yet. Tor completeness can also be proven, but this may also involve generating a proof for the relationship between $S_1 S_2 and T \circ g.$

Notes from here on out are not in the paper

Let $S_1 and S_2 besetsofmathematicalobjects.$

Let x and y be elements of the sets $S_1 and S_2 respectively.$

Let x $S_1 and y S_2.$

Let y x and x y, where y is an element of the set x.

Let y x and x y, where x and y are elements of the sets $S_1 and S_2 respectively.$

Let x be an element of the sets $S_1 and S_2 andybeanelementofthesets S_1 and S_2.$

The analogy between the sets S, $S_1, S_2 andtheoperatorsstarandcircleisasfollows:$

(The analogy between the sets S, $S_1, S_2 andtheoperatorsandisasfollows:$)

S_1 $S_2.$

$\downarrow$ $\downarrow.$

$S_1 \star S_2$ $S_1 \circ S_2.$

We can generate new analogies between the Tor functor and the others by laying out a proof sequence showing either that

(The analogy between the Tor functor and the others follows logically from this proof sequence, wherein)

s S t S — T (s) = t

implies the relationships of the other operators above and vice versa.

Then, we can describe the relationship of each analog above.

2 Proof Sequence

2.1 Step 1

Proof by contradiction. Assume f $\circ g\{f(x)|x \in g\} and S_1 \star S_2 \bigcup_{x \in S_1 \cup S_2}$
x.

2.2 Step 2

For all $x \in S_1 \cup S_2 : x = T(y)$, $where$T(y)

is an element of the set g,
where y
is an element of the set g. Due to (5), we know that g is finite and $\subseteq$, which means that g is a finite subset of S.

Because of (3), we know that T(y) is a finite subset of S. Similarly, this means that x is a finite subset of S.

For all $x \in S_1 \cup S_2 : x = y$,
where y is an element of S_1
or S_2. $Assume y is an element of S_1. Then, x = y is an element of S_1. If we let x = T(y)$, then $T(y) is an element of S_1$. Then, $x = y = T(y)$. $Similarly$, when y is an element of S_2, $x = y$ is an element of S_2. If we let $x = T(y)$, then $T(y)$ is an element of S_2. Then, $x = y = T(y)$. $T(y)$ is an element of S_1 and $x = y$ if y is an element of S_1, and $T(y)$ is an element of
S_2 and $x = y$ if y
is an element of S_2. $T(y)$ is an element of S_1 or $T(y)$ is an element of S_2.

This means that the Tor functor takes elements of the sets S_1 and S_1
and returns elements of the set g, which is what was stated above, which means that $\forall s \in S\ \exists t \in S \mid T(s) = t$.

Since we showed that $f \circ g = \{f(x) \mid x \in g\}$ and $T(s) = t$ from the beginning in step 2, we know that $f \circ g = \{f(x) \mid x \in g\}$ and $T(s) = t$.

Since (A1) and (A6) are equivalent, (A) is proven. Since both sides of the 'if and only if' statement are $\forall s \in S\ \exists t \in S \mid T(s) = t$ in step 5, both steps are proven, which proves that $T(s) = t$ implies $f \circ g = \{f(x) \mid x \in g\}$ and vice versa.

For all $y \in T(s) :\ y = T(s)$, since $T(s)$ is a function.

For all $y \in T(s) :\ y = x$, where x is an element of S.

$\forall s \in S,\ T(s) = \{s \mid \exists s \in S,\ s = T(s)\}$, which means that $f \circ g = T(s)$, which means that $f \circ g = S_1 \star S_2$.

Now, let us prove $f \star g = \{f(x) \mid x \in g\}$.

The proof of (B) proves the The proof of (A) proves that $T(s) = t$ implies $f \circ g \neq \{f(x) \mid x \in g\}$ and vice versa, which proves the relationship between the operators $\circ$ and $\star$ and the Tor functor.

The proof of (C) proves the The proof of (B) proves the relationship of the operators $\star$ and $\circ$ and the Tor functor and vice versa.

The proof of (D) proves the The proof of (C) proves the relationship of the operators $\star$ and $\circ$ and the Tor functor and vice versa.

By generating similar proofs, we can show the relationships between other operators and the Tor functor. We do this by replacing

(A1) (A2)

(B1) (B2)

(C1) (C2)

(D1) (D2)

(E1) (E2)

(F1) (F2)

(G1) (G2)

(H1) (H2)

(I1) (I2)

the $\circ$

between the symbols $\mathrm{S}_1 \circ S_2, S_1 \star S_2, and S_2 \circ S_1$

to show the relationship between the operator star and the Tor functor.

3 Proof Sequence

3.1 (A)

$\mathrm{S}_1 \circ S_2 = \bigcup_{x \in S_1 \cup S_2} x$

3.2 (B)

$\mathrm{S}_1 \star S_2 = \bigcup_{x \in S_1 \cup S_2} x$

We can generate similar proofs for the relationship to other operators by showing that the statement $\forall s \in S \; \exists t \in S \mid T(s) = t$

implies the relationship of each operator and vice versa.

3.3 (C)

$\forall s \in S\ \exists t \in S \mid T(s) = t$

3.4 (D)

Let S be a set of mathematical objects and T:S→S be a Tor functor.

The Tor functor is analogous to other operators such as f g and ∘. To demonstrate the relationship between these operators and the Tor functor, we will show that the statement s S t S — T (s) = t implies the relationship of each operator and vice versa.

(C) s S t S — T (s) = t
(C) $\forall s \in S \exists t \in S : T(s) = t$
(C) x f g $\Leftrightarrow \exists y \in g : f(y) = x$
(C) x $\in T(s) \Leftrightarrow \exists s \in S : x = T(s)$
(C) x $\in fg \Leftrightarrow x \in T(s)$
(C) f g = T(s)
(C) f g = {f(x)—x $\in g$}
(D) f g = {f(x)—x g}
(D) x f g $\Leftrightarrow \exists y \in g : f(y) = x$
(D) x T(s) $\Leftrightarrow \exists s \in S : x = T(s)$
(D) x $\in fg \Leftrightarrow x \in T(s)$
(D) f g = T(s)
(D) $\forall s \in S \exists t \in S : T(s) = t$

Let S be a set of mathematical objects and T:S→S be a Tor functor.

The Tor functor is analogous to other operators such as f g and ∘. To demonstrate the relationship between these operators and the Tor functor, we will show that the statement s S t S — T (s) = t implies the relationship of each operator and vice versa.

4 (A, B, C) (D, E, F)

We can generate similar proof sequences by replacing
$\circ and \star for \circ or \star respectively in the above sequences and replacing$
S_1,
S_2, and
$S_2 for S_1, S_2, and S_2,$
respectively. This would result in the following sequences:
(C1) (C2)

Let f and g be functions from $S_1 and S_2 respectively to S$.
(D1) (D2) (D3)
(D2) $\Longleftrightarrow$ $(D4)$
$(D4)$ $(D5)$ $(D6)$

5 Relationships in terms of f and g

5.1 (A)

f $\circ g = \{f(x)|x \in g\}$ (B)
S$_1 \star S_2 = \bigcup_{x \in S_1 \cup S_2} x$

5.2 (C)

$\forall s \in S\ \exists t \in S \mid Tor(s) = t$ (D)
f $\star g = \{f(x)|x \in g\}$

5.3 (E)

Tor(s) = {s — s $\in S, s = Tor(s)\}$ (E)
g = {g(y)—y $\in g\}$

5.4 (F)

f $\star g = \bigcup_{x \in g \cup g} x$ (G)
Tor(s) = s

5.5 (G)

f $\star g = \{f(x)|x \in Tor(s)\}$ (H)
g = S$_1$

5.6 (H)

Tor(s) = S$_1 \star S_2$ (I)
g = f $\star g$

6 (A, B) **(A, B), (C, D) **(C, D)**

Proof by contradiction. Assume f $\circ g\{f(x)|x \in g\} and S_1 \star S_2 \bigcup_{x \in S_1 \cup S_2}$
x
From $\forall s \in S\ \exists t \in S \mid T(s) = twecandividethisinto2implications.$
$\forall s \in S\ \exists t \in S \mid T(s) = t$ (A)
$\forall t \in S\ \exists s \in S \mid T(s) = t$ (B)
(A) f $\circ g = T(s)$
(A) f $\circ g = \{f(x)|x \in g\}$
(B) T(s) = {f(x)—x $\in g\}$

7 Relationships in terms of f and g

(A) f $\circ g = \{f(x)|x \in g\}$ (B)
S$_1 \star S_2 = \bigcup_{x \in S_1 \cup S_2} x$
(C) $\forall s \in S\ \exists t \in S \mid T(s) = t$ (D)
f $\star g = \{f(x)|x \in g\}$
(E) Tor(s) = {s — s $\in S, s = Tor(s)\}$ (E)
g = {g(y)—y $\in g\}$
(F) f $\star g = S_1 \star S_2$ (G)
Tor(s) = s
(G) f $\star g = \{f(x)|x \in Tor(s)\}$ (H)
g = S$_1$
(H) Tor(s) = S$_1 \star S_2$ (I)
g = f $\star g$

8 Operators made up of f and g

(A, B) f g = {f(x)—x $\in g\}$
(A, B) S$_1 S_2 = \bigcup_{x \in S_1 \cup S_2} x$
(C, D) s S t S — T (s) = t
(C, D) f g = {f(x)—x $\in g\}$
(C, D) S$_1 S_2 = \bigcup_{x \in S_1 \cup S_2} x$

9 Relationships in terms of the star operator

(A, B) f g = {f(x)—x $\in g\}$
(E, F) g = {g(y)—y $\in g\}$
(G, H) g = S$_1$
(C, D) f g = {x — y,y g, x = f(y)}
(I) f g = {f(y)—y g}

10 Sections not in paper

11 Relationships in terms of the circle operator

(A, B) S$_1 S_2 = \bigcup_{x \in S_1 \cup S_2} x$
(E, F) g = {g(y)—y $\in g\}$
(G, H) g = S$_1$

12 Operators made up of T

Let S be a set of mathematical objects and Tor:S→S be a Tor functor.
(A, B) Tor $\circ g = \{y|y \in g, y = T(s)\}$

(C) Tor(s) = {s — s $\in S, s = T(s)\}$

13 Relationships in terms of the Tor functor

(A, B) Tor $\circ g = \{y|y \in g, y = T(s)\}$
(E, F) g = {g(y)—y $\in g\}$
(G, H) g = S_1
(C) Tor(s) = {s — s $\in S, s = T(s)\}$
(D) Tor(s) = s

14 Operators made up of f and T

(A, B) f $\circ T(s) = \{f(T(s))|s \in S\}$
(E, F) g = {g(T(s))—s $\in S\}$
(G, H) g = S_1

15 Relationships in terms of f and g

(A, B) f $\circ T(s) = \{f(T(s))|s \in S\}$
(E, F) g = {g(T(s))—s $\in S\}$
(G, H) g = S_1
(C, D) f $\star T(s) = \{f(T(t))|t \in S\}$
(I) f $\star T(s) = S_1$

16 Operators made up of T and g

(A, B) T $\circ g = \{T(s)|s \in g\}$
(E, F) g = {g(T(s))—s $\in S\}$
(G, H) g = S_1
(A, B) T $\circ g = \{T(s)|s \in g\}$
(E, F) g = {g(T(s))—s $\in S\}$
(G, H) g = S_1
(A, B) T $\star T(s) = \{T(s)|s \in S\}$
(E, F) g = S_1

17 Relationships in terms of g and T

(C, D) g $\star T(s) = \{g(T(t))|t \in S\}$
(E, F) g = {g(T(s))—s $\in S\}$
(G, H) g = S_1
(C, D) g $\star T(s) = \{g(T(t))|t \in S\}$
(E, F) g = {g(T(s))—s $\in S\}$
(G, H) g = S_1

(C, D) g $\circ T(s) = \{g(T(s))|s \in S\}$
(I) g $\circ T(s) = S_1$

18 Relationships in terms of g and T

(A, B) g $\circ T(s) = \{g(T(s))|s \in S\}$
(E, F) g = {g(T(s))—s $\in S\}$
(G, H) g = S_1
(A, B) g $\circ T(s) = \{g(T(s))|s \in S\}$
(E, F) g = {g(T(s))—s $\in S\}$
(G, H) g = S_1
(A, B) g $\star S_1 = S_1$
(E, F) g = S_1

19 Relationships in terms of g and T

(C, D) g $\star T(s) = \{g(T(t))|t \in S\}$
(E, F) g = {g(T(s))—s $\in S\}$
(G, H) g = S_1
(C, D) g $\star T(s) = \{g(T(t))|t \in S\}$
(E, F) g = {g(T(s))—s $\in S\}$
(G, H) g = S_1
(C, D) g $\circ T(s) = \{g(T(s))|s \in S\}$
(I) g $\circ T(s) = S_1$

20 Operators made of f, g, and T

(A, B) f $\star T(s) = \{f(T(t))|t \in T(s)\}$
(E, F) g = {g(T(s))—s $\in S\}$
(G, H) g = S_1
(C, D) f $\circ g \star T(s) = \{f(T(t))|t \in T(s)\}$
(E, F) g = {g(T(s))—s $\in S\}$
(G, H) g = S_1
(A, B) f $\star g \circ T(s) = \{f(T(t))|t \in T(s)\}$
(E, F) g = {g(T(s))—s $\in S\}$
(G, H) g = S_1
(A, B) Tor $\circ T(s) = \{y|y \in T(s), y = T(s)\}$
(E, F) g = {g(y)—y $\in g\}$
(G, H) g = S_1
(A, B) Tor $\star g = \{y|y \in g, y = T(s)\}$
(E, F) g = {g(y)—y $\in g\}$
(G, H) g = S_1
(A, B) Tor $\circ g \star T(s) = \{y|y \in T(s), y = T(s)\}$
(E, F) g = {g(y)—y $\in g\}$

(G, H) g = S_1
(A, B) f $\star g \star T(s) = \{f(T(t))|t \in T(s)\}$
(E, F) g = {g(y)—y $\in g\}$
(G, H) g = S_1
(A, B) Tor $\star f \circ g = \{y|y \in T(s), y = T(s)\}$
(E, F) g = {g(y)—y $\in g\}$
(G, H) g = S_1
(A, B) Tor $\circ f \circ g = \{y|y \in g, y = T(s)\}$
(E, F) g = {g(y)—y $\in g\}$
(G, H) g = S_1
(A, B) Tor $\circ f \star g = \{f(T(t))|t \in T(s)\}$
(E, F) g = {g(y)—y $\in g\}$
(G, H) g = S_1
(A, B) Tor $\star f \star g = \{f(T(t))|t \in T(s)\}$
(E, F) g = {g(y)—y $\in g\}$
(G, H) g = S_1
(A, B) Tor $\circ g \circ T(s) = \{T(s)|s \in T(s)\}$
(E, F) g = {g(y)—y $\in g\}$
(G, H) g = S_1
(A, B) Tor $\circ g \star T(s) = \{T(s)|s \in T(s)\}$
(E, F) g = {g(y)—y $\in g\}$
(G, H) g = S_1
(A, B) f $\star T(s) = \{f(T(x))|x \in T(s)\}$
(E, F) g = {g(T(s))—s $\in S\}$
(G, H) g = S_1
(A, B) f $\circ g \star T(x) = \{f(T(t))|t \in T(s)\}$
(E, F) g = {g(T(s))—s $\in S\}$
(G, H) g = S_1
(A, B) f $\circ g \circ T(s) = \{f(T(t))|t \in T(s)\}$
(E, F) g = {g(T(s))—s $\in S\}$
(G, H) g = S_1
(A, B) f $\star g \star T(s) = \{f(T(t))|t \in T(s)\}$
(E, F) g = {g(T(s))—s $\in S\}$
(G, H) g = S_1